GCE AS/A LEVEL – NEW

2305U10-1

S18-2305U10-1

FURTHER MATHEMATICS – AS unit 1
FURTHER PURE MATHEMATICS A

MONDAY, 14 MAY 2018 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

1. The matrices $\mathbf{A}$ and $\mathbf{B}$ are such that $\mathbf{A} = \begin{bmatrix} 4 & 2 \\ -1 & -3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$.

(a) Explain why $\mathbf{B}$ has no inverse. [1]

(b) (i) Find the inverse of $\mathbf{A}$. [3]

(ii) Hence, find the matrix $\mathbf{X}$, where $\mathbf{AX} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$. [2]

2. Prove, by mathematical induction, that $\sum_{r=1}^{n} r(r+3) = \frac{1}{3}n(n+1)(n+5)$

for all positive integers n. [6]

3. A cubic equation has roots α, β, γ such that

$$\alpha + \beta + \gamma = -9, \qquad \alpha\beta + \beta\gamma + \gamma\alpha = 20, \qquad \alpha\beta\gamma = 0.$$

(a) Find the values of α, β, and γ. [4]

(b) Find the cubic equation with roots 3α, 3β, 3γ.
Give your answer in the form $ax^3 + bx^2 + cx + d = 0$, where a, b, c, d are constants to be determined. [4]

4. A complex number is defined by $z = -3 + 4\mathrm{i}$.

(a) (i) Express z in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $-\pi \leqslant \theta \leqslant \pi$.

(ii) Express $\bar{z}$, the complex conjugate of z, in the form $r(\cos\theta + \mathrm{i}\sin\theta)$. [4]

Another complex number is defined as $w = \sqrt{5}(\cos 2{\cdot}68 + \mathrm{i}\sin 2{\cdot}68)$.

(b) Express zw in the form $r(\cos\theta + \mathrm{i}\sin\theta)$. [3]

5. *(a)* Show that $\frac{2}{n-1} - \frac{2}{n+1}$ can be expressed as $\frac{4}{(n^2-1)}$. [1]

(b) Hence, find an expression for $\sum_{r=2}^{n} \frac{4}{(r^2-1)}$ in the form $\frac{(an+b)(n+c)}{n(n+1)}$,

where a, b, c are integers whose values are to be determined. [6]

(c) Explain why $\sum_{r=1}^{100} \frac{4}{(r^2-1)}$ cannot be calculated. [1]

6. *(a)* Show that $1 - 2i$ is a root of the cubic equation $x^3 + 5x^2 - 9x + 35 = 0$. [3]

(b) Find the other two roots of the equation. [4]

7. The complex number z is represented by the point $P(x, y)$ in the Argand diagram and

$$|z - 4 - i| = |z + 2|.$$

(a) Find the equation of the locus of *P*. [4]

(b) Give a geometric interpretation of the locus of *P*. [1]

8. The transformation *T* in the plane consists of a translation in which the point (x, y) is transformed to the point $(x - 1, y + 1)$, followed by a reflection in the line $y = x$.

(a) Determine the 3×3 matrix which represents *T*. [4]

(b) Find the equation of the line of fixed points of *T*. [2]

(c) Find T^2 and hence write down T^{-1}. [3]

9. The line L_1 passes through the points $A(1, 2, -3)$ and $B(-2, 1, 0)$.

(a) (i) Show that the vector equation of L_1 can be written as

$$\mathbf{r} = (1 - 3\lambda)\mathbf{i} + (2 - \lambda)\mathbf{j} + (-3 + 3\lambda)\mathbf{k}.$$

(ii) Write down the equation of L_1 in Cartesian form. [4]

The vector equation of the line L_2 is given by $\mathbf{r} = 2\mathbf{i} - 4\mathbf{j} + \mu(4\mathbf{j} + 7\mathbf{k})$.

(b) Show that L_1 and L_2 do not intersect. [5]

(c) Find a vector in the direction of the common perpendicular to L_1 and L_2. [5]

END OF PAPER

